General Instructions: Use black or blue pen only. Show neat, complete and organized solutions to earn full points. Box all final answers. The use of any electronic devices is not allowed during the exam. Cheating is punishable by a grade of **5.00** for the course.

I. Let

$$f(x) = \begin{cases} \dfrac{x^2 - x}{x} & , x < 1, x \neq 0 \\ \left[\!\left[\dfrac{x}{3} \right]\!\right] & , 1 \leq x \leq 5 \\ \dfrac{x - 1}{x^2 - 6x + 5} & , x > 5. \end{cases}$$

List down the **three** points of discontinuity of f, identify each as a removable discontinuity, a jump discontinuity or an infinite discontinuity and evaluate the appropriate limits to back up your answers.

II. Evaluate $\displaystyle\lim_{x \to 4^-} \frac{|10 - 3x| - [\![x - 1]\!]}{x - 4}$.

III. Evaluate $\displaystyle\lim_{x \to -\infty} \sqrt{x^2 + 3x} + x$.

IV. Evaluate $\displaystyle\lim_{x \to \frac{3\pi}{4}} \frac{2\cos x + \sqrt{2}}{4x - 3\pi}$.

V. Let g be a function defined on an open interval such that $x \sin\left(\dfrac{3}{x}\right) \leq g(x) \leq \dfrac{3x^4 + 16x^2 + 5}{x^4 + 6x^2 + 5}$. Evaluate $\displaystyle\lim_{x \to \infty} g(x)$.

VI. Consider the following functions defined on the interval $[-1, 2]$.

- $f(x) = \begin{cases} 2x - 1 & , x < 0 \\ x^2 + 2 & , x \geq 0. \end{cases}$
- $g(x) = x^3 - 4x^2 - 3x + 18.$
- $h(x) = x^4 + 2x^3 - 3x - 6.$

Explain why we can **not** use the Intermediate Zero Theorem on f and g on the interval $[-1, 2]$. Then, use the Intermediate Zero Theorem to show that h as at least one zero on the interval $[-1, 2]$.

END OF EXAM

TOTAL: 12 POINTS

guissmo